THH & the description of THH(F) ~ Some to Böksbedt (unpublished), also dare by Breen puely algebraically -> Still it seems that THH (Fp) & as a vity, one needs some toyological reput, e.g. Steemrod operations. Sec Franjon-Lamer-Schwartz. For a reference, Blurbag - Cohen - Schlictkoull a Thom spectrum

See also notes by Krave-Nikolans for a discussion. (on THH).

Topological cochic homology appeared In Bölestedt-Høring - Madsen $K(R) \longrightarrow F(CR)$

Theorem of Dundas-Goodwillie-AcCarthy if Raving, JCR hilpotent, have a cartesian square $K(A) \rightarrow TC(R)$ $K(R/E) \rightarrow TC(R/E)$

Applications by Hesselholt, Madsen to compute K-theory of 10ts of ving S.

-> See Madsen's showing traces" "Algebraic It-theory traces" Cyclotomic specta TC(R) = Hom (1, THH(R)) CycSp

Initially one didn't have Coc Sp as a honotopy theory. (instead 7C defined more explicitly). Montioned by Kaledin, 2010-

Thes thereastrophy theory Cyc Sp was constructed by Blumberg - Marlel & formula above is correct. Also described by Barrick-Glashon Ayala-Mazel-Gee - Rozenblym

Nikolans-Schobe: all descriptions of cyclotonic spectra have redundancy in bounded - below case. They give a description of cyclodomic Spectra, P X^tG S' X - S X^tG S'/G

The description of THH(R), R a coming of from their pages.

Antican-Nikolans: Give a description of cyclotomic spectra in tems of the maint TR ~> "topological Cartier modules"

Main theorem (Bhatt-Momon-Scholge) R smooth algebra / perfect field & Than there is a filtration on the TP(R) = THH(R) whose associated graded is

(2-periodic) cystalline cohomology of R.

Ex) R = IFp. THH(FFp) = FFp [0], 101=2-

 $TP(F_{p}) = Z_{p} [x] |x] = 2.$

cystalline cold of Feis Zp -

Rink R ang Fip-algebra TP(R) is a mobile over TP(AP) TP(B) FFP ~ HP(R/FF)

Constructing this filtration is somewhat subtle (Antienn-Nikokans gare a simpler ponstrution), BMS really want to imput mixed charmgs (e.g. 24). How to construct filtration on TP(B)? 1) Daire functors so can apply to large Ap-alg- (duire differential fams, dR col.). 2) Construct the filtration directly for large Hp-alg ("regular fect") (Postniker filtration).

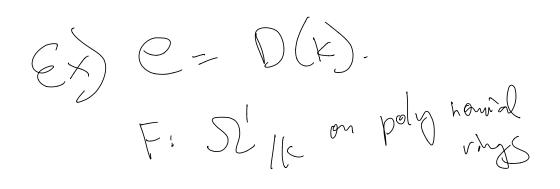
TP is here in even degrees. 3) Define on smooth Ffy-algs by Faithfully flat descent. Derived functors ("nonabelian")-EX) IN field Symi: Vectr -> Vectr. Admits a devied functor IL Symi: D(K)=> D(F)=0. Explicitly P. ~ Dold-Kan into a simplicial k-vector pre > Consider Symip, as a new simplicial

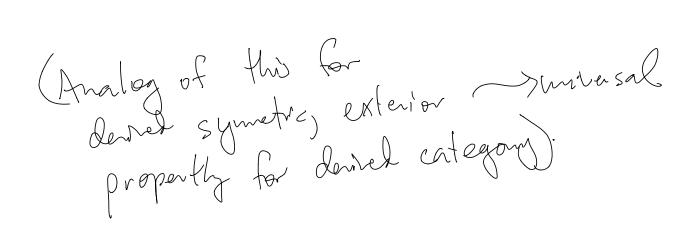
k-vector spenc > apply DK to a chain cx, This is USymi-EX Quillen cotangent complex). Ka base field. Each com kalg R, ~ Str/k Kähler differentig The "derived function" is the cotangent complet LR/K.

Construction: R a ving, choose a simplicial k-alg P. ω P, -> R 2-iso.

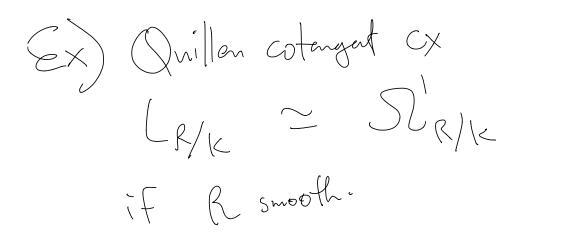
 $L_{R/K} = | S_{P/K}^{\prime} |$ The geometric realization-More generally R can be a SCR. Key propertly: Ho(LR/K) = SCR/K Can be higher tromology ~) does not happen if Q/K smooth. Consider SCRK ~ honotopy theory of simplicial com vitys $Poly_{L}^{f.g.} \subseteq SCR_{k}$

Fact: If C is any on-category w sifted colimits, $F_{m}(Poh_{k}^{f\cdot g'}, e) = F_{m}(SCR_{k}, e)$ commute af sifted colourts -



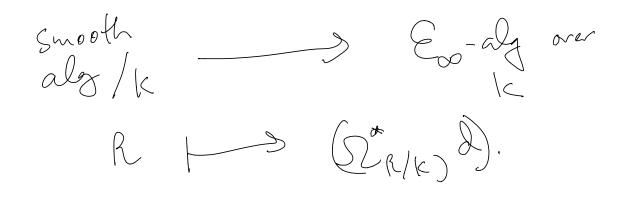


In general, it's hand to make this explicit on vigs (which are not phynomial).



Another example: Derived de Rhan cohonology. Recall (ka base field) R/k alg (smooth),

Consider (2° k) b) ~ commutate den over k.



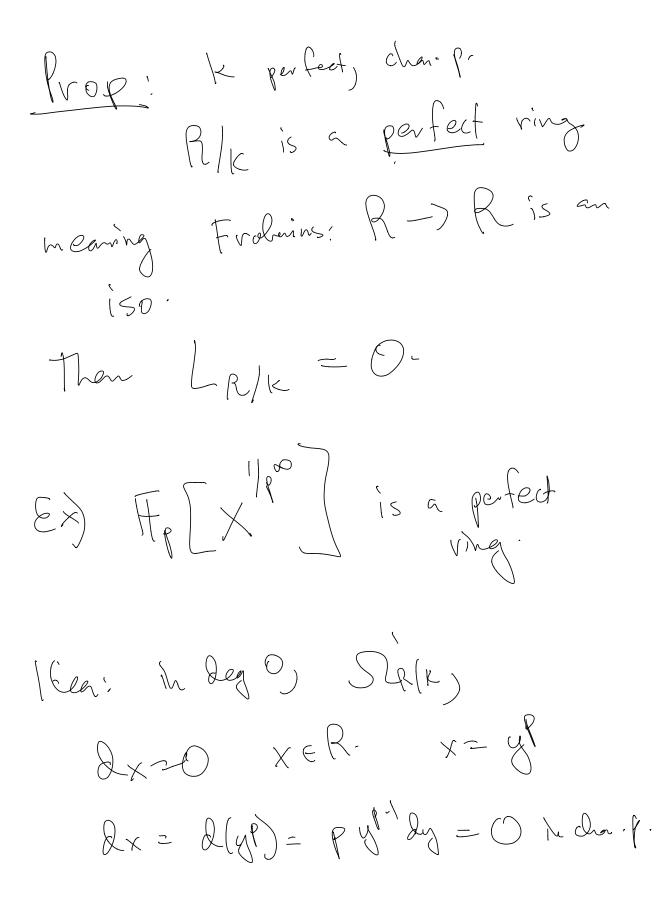
(et's try to deve this construction (Illusie). Restrict polynomial rings & then vesolve (simplicial) ungs by polynomial mgs. 3 Eo-alg over k QRK: SCRK

"Denived de Rham cohomology" Ex) IF R is a pay ving) it's usual de Rham complex) it's usual de Rham complex) it general based on some verolution. Theorem (Bhatt): k perfect, chen. P. DRRIK (Strik) Dif DRRIK R smooth. (Cantion isomorphism Conjugate Filtradia). Not the in char. O'.

Also a theory of derived crystalline cohomology, agres af ord. cystalline coh on swooth algs (rela)-Krop k base field, R/k alg Claim is that HH(R/k) has a (comegeit) lescendry filtration whole gri = $\left(\begin{array}{c} c \\ R \end{array} \right) \left(\begin{array}{c} R \end{array}$ - consider LR/k is in D(R) - A' is ith exterior in R-modules (doired)

Pf: Consider $HH(/k): S(R_k) \longrightarrow D(k).$ this commentes ul sifted Obseme that colimits -Everything is determined by polynomial Vilez S. In fact, HH(1k) is completely by value on polynomial rings. If Pisa polynomial ving) HKR Heoren: HH (P/K) ~ SZP/K (true for smooth algebras).

Take the Postilier filtration Fi HH(P/E) = Tzi HH(P/E) $gr^{\epsilon} = (\Lambda^{i} \Omega_{P/K}) [i]^{\epsilon}$ Proved propries Pris polynomial, now extend family (Kan extension) to get the statement in general. A. For this to be useful, need vings for which LR/K Known



Cor: R/k perfect mg HA(R/K) * = R. (in deg 0)-Pf: Use HKR filtation from previous prop. LRIK = 0. D. Car. $THH(R)_{\alpha} = R[\sigma].$ (L perfect Desuike THH(Fp) ~ Zp[X,] $|\theta| = 2$ $|x| = -2 \quad (x \in H^2(\mathbb{Q}^{p^n}))$

Nikolang-Schobe >> JHH (Fg w a cyclotonic spectrum, tG THH(Fp) ~ T>0 (GP) as Epring