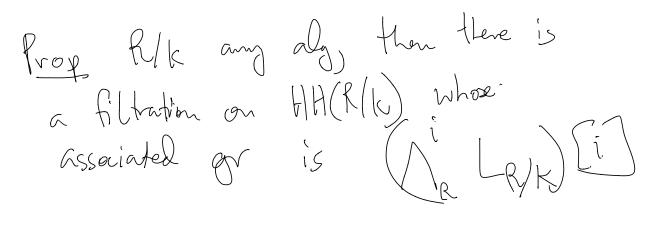
Ingredients in the proof of this theorem

last time, talked about machinery of deviced functors & device de Rhan cohouslogy, cotangent complex L/K. QR (K



~ Pf: Postniker filtration for R smooth daved finctors anywhere.

EX) HH(R/R) = R if K perfect F, ng (k perfectifield). chan p-Sou If R is a perfect Fp-alg  $THH(R)_{\star} = R[\sigma], |\sigma| = 2.$  $(b|c thh(R)/\sigma \simeq Hh(R/F_p) = R)$ Theorem (BMS): The construction R (R) THH(R) satisfies faithfully flat descent

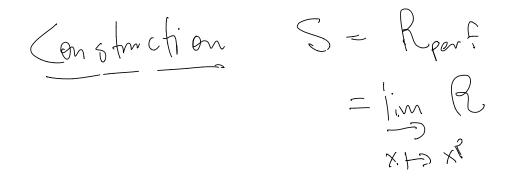
as does RH> TP(R) TC(R) = THH(R)

More precisely, if R - > S, then  $TP(R) \simeq Tot (TP(S) = TP(SQS) = ...)$ Runk: In étale case, Weibel-Geller, McCarthy-Minasian

(Dea: (Bhatt) First prove this for LIFp) here use cofiber sequences for L/Fp

(if A > B = 7C are maps of mass cofiber seq LB/A & C > LC/A > LC/B) & then bootstrap to A<sup>i</sup> L/Ap

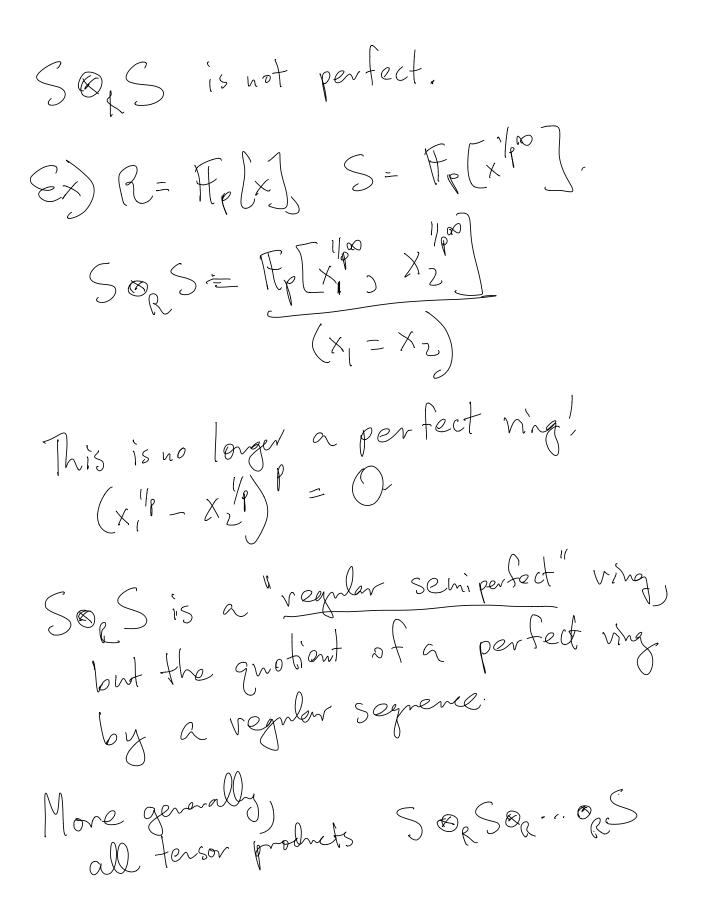
& show that this is a sherp for flat topology-& use HKR filtration ~ HH (/Fp) is also sheaf for flat topology.  $X THH(R)/\sigma = HH(R/Mp)$ STHH(R) a sterf. TP(R) a sheaf (connectivity argument) R smooth Flp-alg. Idea is to use sheaf property to Understand TP(R).

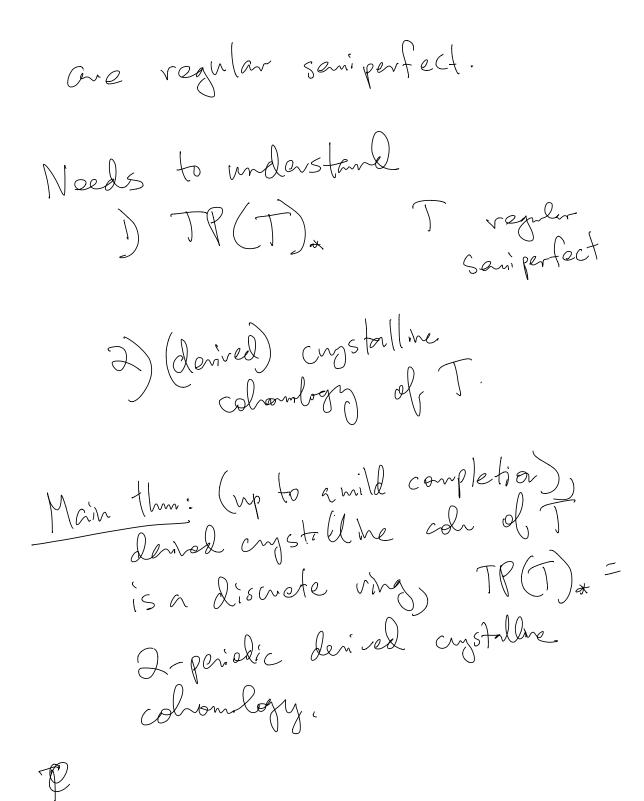


$$\begin{array}{l} \displaystyle \mbox{$\mathbb{E}$x$} \end{array} & \mbox{$\mathbb{R}$} = \mbox{$\mathbb{F}$} p[x] \\ \displaystyle \mbox{$\mathbb{R}$} p p x \mbox{$\mathbb{F}$} = \mbox{$\mathbb{F}$} p[x]^{m} \\ \displaystyle \mbox{$\mathbb{R}$} = \mbox{$\mathbb{F}$} p[x]^{m} \\ \displaystyle \mbox{$\mathbb{R}$} p x \mbox{$\mathbb{F}$} = \mbox{$\mathbb{K}$} \mbox{$\mathbb{F}$} p[x]^{m} \\ \displaystyle \mbox{$\mathbb{R}$} p x \mbox{$\mathbb{F}$} p[x]^{m} \\ \displaystyle \mbox{$\mathbb{R}$} p x \mbox$$

To desube JP(R), it suffices to because  $TP(S), TP(S\otimes_{p}S), \dots$ Iden is that there are easier (even depres).

S perfect T<sub>p</sub>-aly. First case: TP(S)  $\frac{P_{rop}}{P_{rop}} = W(S)[x^{\pm 1}] |x| = -2 -$ Pf.  $THH(S)_{x} = S[\sigma], Jol = Q.$ Run the S'-Tate spectral sequel. Identify the extension. Filtration on TP(S) is the Postniker filtration: (crystallie coll of S B W(S)). Unfortunately, this is not sufficient, Sors, Sorson S. ...

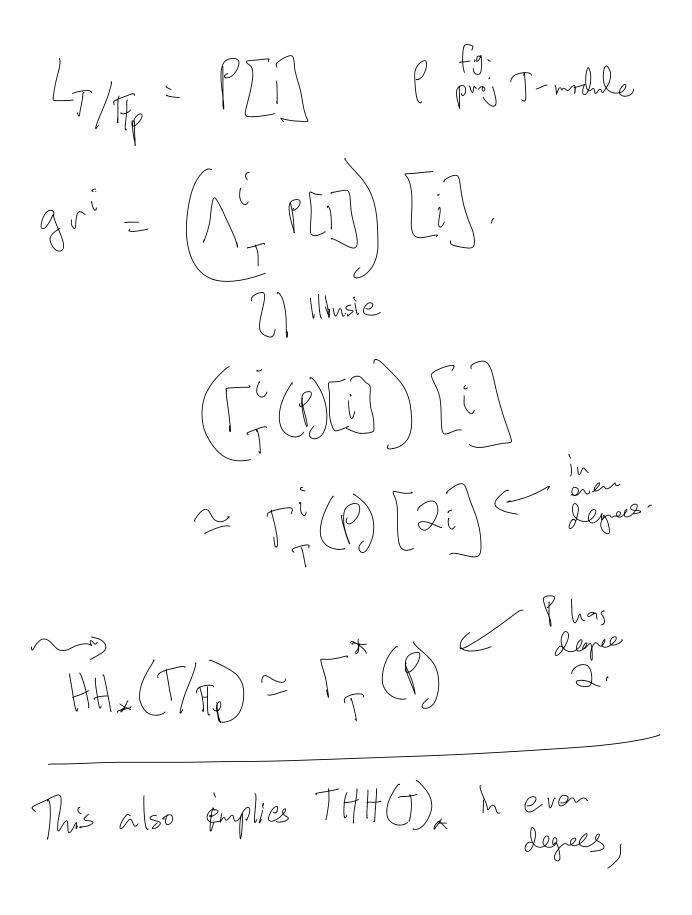




Def the BMS filtration on  $F^{i} TP(R) = Tot(T_{s2i}, TP(S) \neq T_{s2i}, TP(S_{R}S)$ This gives a filtration on TP(R) & the fact that its associated is crystalline con follows from the analogous assortion on TR(SRi)

How do we work of regular service rings T? Claim is that TP, --- and all in even degrees this all boils down. L.T/Hg-

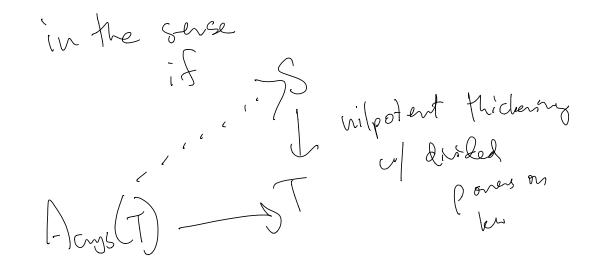
Prop IFT is regler sampenfect, LT/EF is suspension of a f.g. Projectle T-mobile. (in degree 1). Con Tregular seniperfect HH(T/FR) THH(T)~ ave in even degrees. Pf:, Consequence of HKR filtration HH(T/HF) is filtered by gri: (IT/Fp/i)

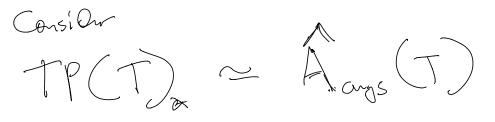


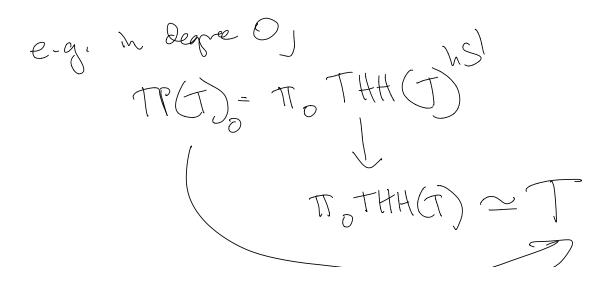
TP(T) is even. Want to say explicitly what TR(T). is, & velate to device crystallare chomology, (BMS) Tregular samiperfect ving) then Lorys (T) (bened crystalline col) is a disaete ving, in degree of S Acrys (T). Moreorer, TPO(T) is the completion of Acrys (I) w.v.t. Nygaard filtration. Deads to BMS filtration.

What is Acros (T)?  $\left( T = \mathcal{F}_{p} \left( X' \right)^{p} \right) \left( X \right)$ , Onstruction of Acrys... Therefore T lin T  $EX (F_{p} [x]^{o})/x \longrightarrow F_{p} [x]^{o} ]$ Consider W(Tperf) DT. O is the unhersal pro-nilpotent (y-adialy complete)

Acrys (J) is the universal (pro) RD thickening of T)







"quasi syntamic site" ZLPET > ZLPET []pool ) S )( R.