Homology and Cohomology
eCHT Winter 2023 Course
MWF 11:00 AM - 11:50 AM ET

Instructor: Sarah Petersen

Teaching Assistant: To be determined

Office Hours: To be determined

Course Objectives: Develop understanding and ease working with algebraic and geometric ideas underpinning homology and cohomology theory including choosing among homology theories, using excision, calculating the cup product, and Poincare Duality for manifolds.

Prerequisites:

- **Rings and Modules**: Solid understanding of groups, rings, fields, homomorphisms, kernels and cokernels
- **Topology**: Familiarity with topological spaces, connectedness, compactness, classification of compact surfaces, covering spaces
- **Topology and Groups**: Homotopic maps, homotopy equivalences

Course Format: We will meet on Zoom on Mondays, Wednesdays, and Fridays from 11:00 - 11:50 AM eastern time. To facilitate active learning, a typical class will consist of approximately twenty minutes of lecture followed by about twenty minutes of working through worksheets in small groups to expand on and further develop ideas covered in the short lecture. The last ten minutes of class will be used to collect and wrap up the main ideas.

Homework: Problem sets will be posted weekly and due at the start of class on Fridays (11 am ET), excepting weeks when course projects are due or exams take place (i.e. no problem sets during Weeks 8, 9 14 and 15). Both neatly handwritten (scanned to pdf) and typed solutions solutions will be accepted. Submission, feedback, and grading will take place on Gradescope.

Projects: Students will complete two expository projects over the course of the semester. The goal of these projects is to develop video presentation and mathematical writing skills.

Video project: Create a short 3-5 minute video explaining a topic covered in the first half of the course. A topic should be chosen by Friday, February 24, a draft outline of the mathematical content uploaded to Gradescope for feedback on Friday, March 3, and the video uploaded to Gradescope by Friday, March 10.

Writing project: Produce a short, self contained reference describing any topic covered in or directly related to the course (distinct from your choice for the video project). This expository write up should be readable by a peer learning the same material for the first time. Submit choice of topic by Friday, April 14, a written draft outline for feedback on Friday, April 21, and the final version submitted on Friday, April 28.

Exams: There will be two exams. The first is a week long take home midterm, which will be due Friday, March 17 at 11 am ET. This exam will take the place of homework for that week. Unlike homework, the midterm is to be completed individually without consulting others. The second exam will be a 50 minute final, which will take place synchronously during class on Friday, May 5.

Technology:

- **Requirements**: Hardware such as a PC, Mac, Chromebook or tablet with speakers, microphone, and webcam allowing for a good Zoom connection
- **Suggestions**: Useful for in class group work: tablet with ability to annotate shared screen. Alternatively, a phone or webcam camera together with pen and paper

Textbooks:

- **Course Book**: Hatcher, Algebraic Topology - Ch 2 & 3
- **Other Books**:
  - **Bredon**, Topology and Geometry
  - **Davis & Kirk**, Lecture Notes in Algebraic Topology
  - **Fulton**, Algebraic Topology: a first course
  - **Massey**, A basic course in Algebraic Topology
  - **May**, A concise course in Algebraic Topology
  - **Munkres**, Elements of Algebraic Topology
  - **Spanier**, Algebraic Topology

Course Topics:

- **Geometric and categorical motivation**: some basic category theory, the fuctors H_* and H^*
- **Δ-complexes and simplicial homology**: n-simplices, Δ-complexes, simplicial complexes, triangulation of simplicial complexes, $H_*^{\Delta}(-)$
- **Singular Homology**: motivation, singlular chain complex, naturality/functorality, $H_*(pt)$
- **Chain homotopies and homotopy invariance**: pairs of spaces, relative homology, LES for a pair, reduced homology $\tilde{H}_*(-)$, LES for $\tilde{H}_*$, naturality of LES for pairs
- **Excision and quotients**: retractions, deformation retractions, good pairs, orientations, excision and Mayer-Vietoris, cone CX suspension ΣX, connected sum $X\#Y$
- **Degree of maps of spheres** tangent vector fields on spheres, hairyball theorom, local degree, proof of fundamental theorem of algebra
- **Cellular homology**: cellular complex, $H_*^{CW}(-)$, rank$H_n^{CW}(X)$

- **Cohomology**: cochains, cohomology $H^*(-)$, functoriality, homotopy invariance, cochain homotopy, dual of SES, excision, LES, Mayer-Vietoris for H^*, Eilenberg-Steenrod axioms
- **Cup product**: $H^*(X)$ unital graded-communative ring, intersection theory
- **Künneth Theorem and Products of Spaces**: tensor product of chain complexes, algebraic Künneth theorem, Euler characteristic, CW-complex for product space
- **Universal Coefficients Theorem**: Ext groups and free resolutions, (co)homology with coefficients in a ring/field/module, universal coefficient theorem for a PID
- **Poincaré Duality for Manifolds**: local orientation, orientation, fundamental class, degree, locally finite homology, cohomology with compact support, cap product, Alexander duality

Meeting Dates:

Week One: January 18, 20
Week Two: January 23, 25, 27
Week Three:: January 30, February 1, 3
Week Four: February 6, 8, 10
Week Five: February 13, 15, 17
Week Six: February 20, 22, 24
Week Seven: February 27, March 1, 3
Week Eight: March 6, 8, 10
Week Nine: March 13, 15, 17

Break: March 20, 22, 24

Week Ten: March 27, 29, 31
Week Eleven: April 3, 5, 7
Week Twelve April 10, 12, 14
Week Thirteen April 17, 19, 21
Week Fourteen April 24, 26, 28
Week Fifteen May 1, 3, 5

Grading:
Homework 20 %
Participation 25 %
Video Project 10 %
Writing Project 15 %
Midterm 15 %
Final 15 %

Extra Help: Do not hesitate to drop by during office hours (on Zoom) or make an appointment with either the instructor or teaching assistant to discuss a homework problem or any aspect of the course.

Attendance Policy: Attendance and participation is expected at every class meeting. Contribution to work in small groups during class directly contributes to grade. Email instructor for up to three excused absences.

Important Dates:

Homework due	Weekly, Fridays 11 am ET
Topic for video project	Friday, February 24, 11 am ET
Outline for video project	Friday, March 3, 11 am ET
Video project	Friday, March 10, 11 am ET
Midterm exam	Friday, March 17, 11 am ET
Topic for writing project	Friday, April 14, 11 am ET
Outline for writing project	Friday, April 21, 11 am ET
Writing project	Friday, April 28, 11 am ET
Final exam	Friday, May 5, 11 - 11:50 am ET

Academic Honesty (*adapted from UC Berkeley's Center for Teaching and Learning*):

Homework and exams are presumed to be your own original work that has not previously been submitted for credit in another course unless you obtain prior written approval the instructor. You may use ideas by other individuals, but only with proper citation and correct attribution to the source (book, publication, web site, person, etc.).

Collaboration and Independence: Working on homework problems, reviewing lecture materials, and studying for exams with peers is encouraged. While discussing homework assignments with peers is encouraged, solutions must be written up individually and the names of everyone who discussed the problem together listed.

Cheating, Plagiarism, and Self-plagiarism: You are expected to write your own solutions and to provide full and accurate citations for any ideas from other sources, including any of your own previously or concurrently submitted course work. Cheating and failure to provide citations may result in a failing grade.