Instructor: Christy Hazel
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Teaching Assistant: TBD
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Office Hours: TBD

Class Meetings: 12-12:50pm Eastern time TuTh from January 17th–April 25th.

Course Description: This course serves as an introduction to topological K-theory. Since its introduction by Atiyah and Hirzebruch in 1961, K-theory has become an essential tool in algebraic topology. In this course we'll cover vector bundles, topological K-theory as a generalized cohomology theory, and Bott periodicity. We will also see applications to classical problems such as the Hopf invariant one problem and vector fields on spheres.

Course Format: We will meet on Zoom 12-12:50pm Eastern Time on Tuesdays and Thursdays. Each Tuesday a student will give a presentation on our weekly topic. Each Thursday students will do work on a short problem set in breakout rooms. It is also expected students will complete weekly readings and work on problems outside of class meetings.

Prerequisites: Students should have a solid understanding of the main topics from Chapters 2–4 from Hatcher's Algebraic Topology (homology, cohomology, and higher homotopy groups).

Technology:

- *Required*: Hardware to access and participate in Zoom meetings such as a PC, Mac, Chromebook or tablet with speakers, microphone, and webcam. You should have a set up that allows for a good Zoom connection.
- *Suggested*: For group work, it's recommended to have a tablet or alternatively a phone set up with paper and pens.

Main References: *A geometric introduction to K-theory* by Dan Dugger. Available on the author's website here.
Vector bundles and K-theory by Allen Hatcher. Available on the author's website here.

Grading: Course grades will be weighted according to the following scheme:

Presentation	50%
Weekly Group Problem Submissions	20%
Participation in Weekly Group Work	15%
Attendance during Presentations	15%

Presentations: Each student will give one 50 minute presentation. A detailed scheduled with recommend resources will be provided. It is unlikely you will be able to cover all of the details during your presentation. You should instead plan to give an overview of the key ideas. It will be expected that other students will complete readings to fill in the details.

Weekly Group Work: Each week there will be 3–4 homework problems associated to the weekly topic. At the start of the week, you will be given one problem that you should prepare a solution for by Thursday. At the start of Thursday, you will be placed into a breakout room for about 15 minutes with people who have the same problem as you to confirm your answers and check your work. Then for the remaining 30 minutes, you'll be placed into a group with 3–4 students who all have different problems. You'll each take turns presenting your problems.

Weekly Group Problem Submissions: Each group will submit one solution set by the end of Friday. Groups can create and edit a shared document using Overleaf. You will be responsible for typing the solution to your problem in the shared document as well as reading your group members' solutions and giving any feedback before submission. One person will submit the group assignment to Gradescope.

Attendance and Participation: It is expected that all students will attend and participate in all class meetings. Part of your grade is dependent on your engagement during presentations and your participation during weekly group work. If you are unable to attend a presentation session or a group work session, then you need to email the instructor or to the TA so that we can plan accordingly (this is especially important for group work because your group will be counting on you to provide a solution for your problem).

Readings and Independent Study: It is expected that all students will read and review the course topics each week. Presentations will not be able to cover the full details of each topic, so students should reserve a few hours each week to go through readings.

Class Schedule:

Week 1 (Jan 17, 19): Motivation and first definitions
Week 2 (Jan 24, 26): Vector bundle operations and classifying vector bundles
Week 3 (Jan 31, Feb 2): Vector bundles on spheres and clutching constructions
Week 4 (Feb 7, 9): $\mathrm{Vect}_n(X)$ and K-theory
Week 5 (Feb 14, 16): K-theory as a generalized cohomology theory
Week 6 (Feb 21, 23): K-theory of spheres
Week 7 (Feb 28, Mar 2): Recap and review

Break (Mar 7, 9)

Week 8 (Mar 14, 16): Bott Periodicity
Week 9 (Mar 21, 23): Bott Periodicity, continued
Week 10 (Mar 28, 30): Adams operations
Week 11 (Apr 4, 6): The Hopf invariant and division algebras
Week 12 (Apr 11, 13): Vector fields on spheres and Clifford Algebras
Week 13 (Apr 18, 20): Vector fields on spheres and Clifford Algebras, continued
Week 14 (Apr 25): Conclusions and sewing up loose ends