STABLE HOMOTOPY THEORY
Spring 2024

COURSE DESCRIPTION

This course is an introduction to stable homotopy theory. The basic objects of stable homotopy theory are called *spectra*, and the exact definition of a spectrum depends on who you ask, what they care about, and even when you ask them. Nevertheless, concepts from stable homotopy theory have found many applications both within algebraic topology and in other fields like geometric topology, algebraic geometry, number theory, algebra, etc. The goal of this course is that you'll be able to work with spectra in a practical manner and perform computations – you can choose your favorite definition later.

LEARNING OUTCOMES

After taking this course, students will be able to engage in research projects in stable homotopy theory or related fields (such as algebraic geometry or geometric topology) that use stable homotopy techniques.

TOPICS

A potential list of topics includes:

- Spectra
- Stable homotopy groups
- Extraordinary homology and cohomology theories
- Stable homotopy category
- Constructions of the stable homotopy category
- Smash product
- Associative and commutative ring spectra
- Spectral sequences
- Operads
- Homotopy limits and colimits
- Spectra and manifold theory

The function between topics covered in the course and the list above may be neither injective nor surjective.

REFERENCES

We will draw from a variety of sources, including but not limited to:

(1) Malkiewich, Spectra and Stable Homotopy Theory (unfinished draft)
(2) Barnes–Roitzheim, Foundations of Stable Homotopy Theory
(3) Margolis, Spectra and the Steenrod Algebra
(4) Ravenel, Complex Cobordism and Stable Homotopy
(5) Adams, Stable Homotopy and Generalized Homology
(6) Schwede, Symmetric Spectra (unfinished draft)
(7) Blumberg–Gerhardt–Hill (eds.), Stable Categories and Structured Ring Spectra

Students are not required to obtain any of these references.

Prerequisites

- Students should have taken the equivalent of two semesters of algebraic topology at the graduate level, including homology, cohomology, and homotopy groups π_n for $n > 1$. Exposure to K-theory, characteristic classes, cobordism, and other topics in algebraic topology is not required, although it may be helpful.
- Students should have a working knowledge of category theory, including but not limited to: natural transformations, limits, colimits, adjunctions, abelian categories, and symmetric monoidal categories.

Please contact the instructor if you do not meet the prerequisites but would still like to enroll in the course.

Logistics

This course will be taught in a flipped-classroom format, with all course meetings online. Before each course meeting, students are expected to either read from a reference or watch a short video. Classes will consist of a short recap of the material by the instructor, followed by group work on problem sets. These problem sets will be due for a grade at the beginning of the following week.

Contact Information

Instructor: David Mehrle

Email: davidm@uky.edu

Office Hours: TBA

Teaching Assistant: TBA

Email:

Office Hours:

Class Time and Location

Dates: 16 January 2024 to 30 April 2024

Break: No class meetings on 26 March 2024 and 28 March 2024

Meeting Time: Tuesdays and Thursdays, 12pm to 12:50pm Eastern U.S. Time

Classroom Meeting Link: TBA

Technology Requirements:

- Hardware to access and participate in Zoom meetings, such as a computer, phone, or tablet with speakers, microphone, and webcam.
- A tablet and pen with the capability of sharing digital handwriting.
- A reliable connection to the internet that does not interrupt your participation in the course.

If you anticipate having any trouble meeting the technology requirements, please contact the instructor.

Grades

As a general rule, students are expected to obtain 2-3 independent study credits from their home universities for their participation in the course. At the end of the semester, a grade of A/B/C/Fail will be reported to your home institution. Grades will be based on a combination of participation, problem sets, presentations, and a take-home final exam. *An exact grading scale will be announced at the beginning of the semester.*