DIFFERENTIALS IN THE ρ -BOCKSTEIN SPECTRAL SEQUENCE

EVA BELMONT AND DANIEL C. ISAKSEN

1. ρ -Bockstein table

The accompanying table displays differentials in the ρ -Bockstein spectral sequence

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$$\operatorname{Ext}_{\mathcal{A}^{\mathbb{C}}}^{***}(\mathbb{M}_{2}^{\mathbb{C}},\mathbb{M}_{2}^{\mathbb{C}})[\rho] \implies \operatorname{Ext}_{\mathcal{A}^{\mathbb{R}}}^{***}(\mathbb{M}_{2}^{\mathbb{R}},\mathbb{M}_{2}^{\mathbb{R}})$$

discussed in [1]. It is a more complete version of [1, Table 5]. For an explanation of the general strategy of the computation, see $[1, \S 5]$.

The d_1 differentials are omitted from the table because there is a large number of them, and they are simple to describe: in the range of s + f - w degrees considered in the table, the d_1 differentials are described entirely by multiplicative relations applied to the following d_1 differentials.

source	target	s	f	w
au	$ ho h_0$	0	0	-1
au g	$ ho h_0 g$	20	4	11
$ au g^2$	$\rho h_0 g^2$	40	8	23
$\Delta c_0 d_0$	$\rho h_0 d_0 l$	46	11	25
$\Delta c_0 e_0$	$\rho h_0 e_0 l$	49	11	27

Description of the columns: A Bockstein differential $d_r(x) = \rho^r y$ is recorded by a line in the table as follows.

- \bullet source: x
- target: y
- **s**: stem of x
- **f**: Adams filtration of x
- s-w: coweight of x
- s+f-w: see f, s w
- diff length: r, unless the class is a permanent cycle, in which case this field is 0

Some notes about shorthand employed in the table:

• Names of elements correspond to names in C-motivic Ext. See https://s.wayne.edu/isaksen/files/2020/04/Adamscharts.pdf for the names and degrees of \mathbb{C} -motivic elements. In particular, t means τ and D means Δ .

- If an element has diff length = 0, this means that the element is ρ -local.
- Starting in degree s + f w = 18, the differentials are presented in the order they were computed. (This is relevant because there are many process of elimination arguments used in the computation.)

References

[1] Eva Belmont and Daniel C. Isaksen, R-motivic stable stems (2020), preprint, available at arXiv:2001.03606.