How Can You Add Numbers Forever and Ever and Ever?
Infinite Series

Presenters:

Shauda Islam
Israel Cleaves
Torianno Thomas II
MyKah Allen
Amaya Muhammad-Richardson
Christian McQueen
Omaria Brooks
Dwight Griffin
Jaleigh Foster
Nyla Mersier
Brooklynn Smiley

Infinite Series

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$$\sum_{i=1}^{\infty} 2^{-i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$$

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The genius that solved this WEIRD problem – Sir Isaac Newton

Infinite Series - How to Add Forever

$$\sum_{i=1}^{\infty} 2^{-i} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ... = 1$$

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Newton's First Big Idea - Partial Sums

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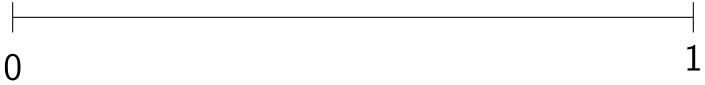

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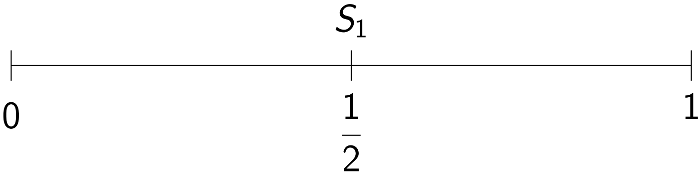

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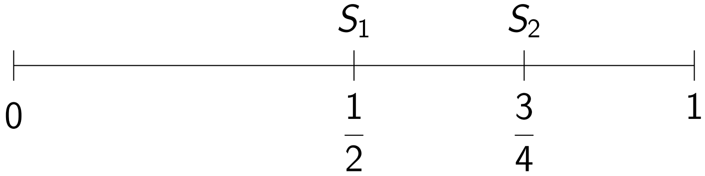

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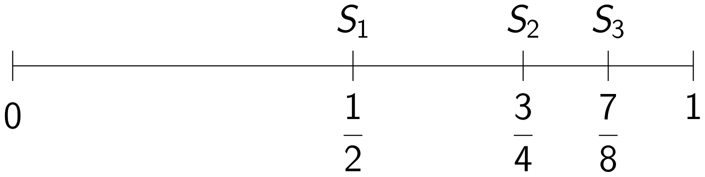

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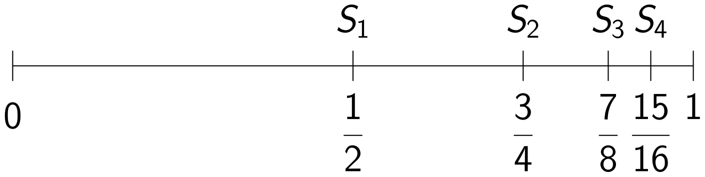

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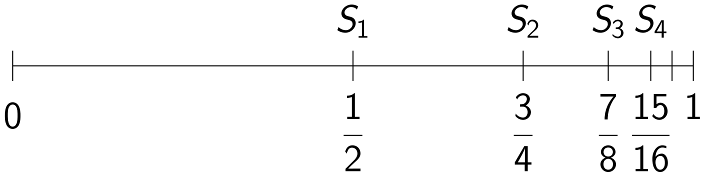

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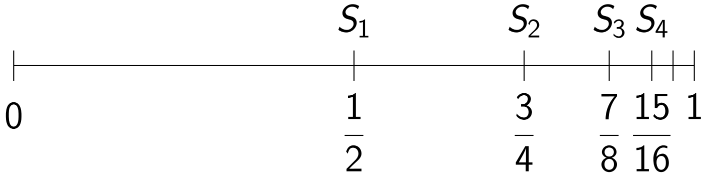

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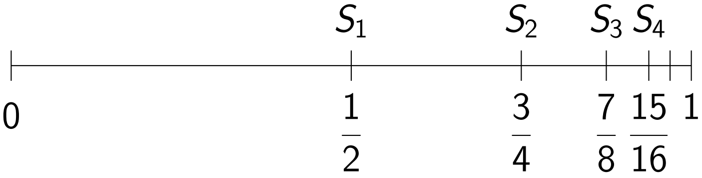
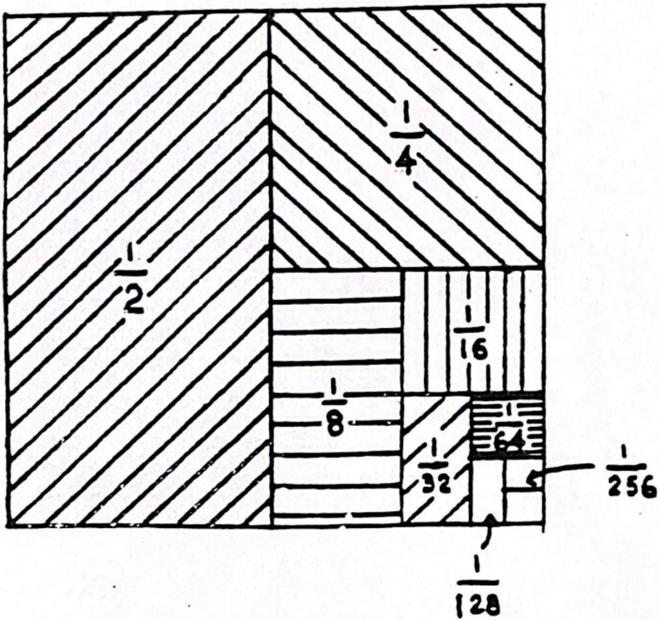


$$S_N = \frac{2^N - 1}{2^N} \text{ for any natural number } N$$

Infinite Series - How to Add Forever



Newton's Second Big Idea - The Limit

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<u>Proof:</u>

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$$= \lim_{N \to \infty} \left(\frac{2^N - 1}{2^N} \right)$$

Newton's Second Big Idea - The Limit

Proof:
$$\sum_{\mathrm{i}=1}^{\infty} 2^{-\mathrm{i}} = \lim_{N\to\infty} S_N$$
$$= \lim_{N\to\infty} \left(\frac{2^N - 1}{2^N} \right)$$
$$= \lim_{N\to\infty} \left(\frac{2^N}{2^N} - \frac{1}{2^N} \right)$$

Newton's Second Big Idea - The Limit

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$$= \lim_{N\to\infty} \left(1 - \frac{1}{2^N} \right)$$

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$$= 1 - \lim_{N\to\infty} \frac{1}{2^N}$$

Newton's Second Big Idea - The Limit

<u>Proof:</u>
$$\begin{aligned}\sum_{i=1}^{\infty} 2^{-i} &= \lim_{N\to\infty} S_N \\ &= \lim_{N\to\infty}\left(\frac{2^N-1}{2^N}\right) \\ &= \lim_{N\to\infty}\left(\frac{2^N}{2^N}-\frac{1}{2^N}\right) \\ &= \lim_{N\to\infty}\left(1-\frac{1}{2^N}\right) \\ &= 1-\lim_{N\to\infty}\frac{1}{2^N} \\ &= 1-0 \\ &= 1\end{aligned}$$

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Proof: $$\sum_{i=1}^{\infty} 2^{-i} = \lim_{N\to\infty} S_N$$

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$$\therefore \sum_{i=1}^{\infty} 2^{-i} = 1$$

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\end{aligned}$$

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Q.E.D.

$$\sum_{i=1}^{\infty} 3^{-i} = \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \frac{1}{243} + \ldots =$$

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$$S_N = \frac{1}{2} \times \frac{3^N - 1}{3^N} \text{ for any natural number } N$$

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$$\sum_{i=1}^{\infty} 4^{-i} = \frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \frac{1}{1,024} + ... =$$

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7	≈ 0.33331

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$$\sum_{i=1}^{\infty} 4^{-i} = \frac{1}{3} \iff \lim_{N\to\infty} S_N = \frac{1}{3}$$

Proof: $\sum_{i=1}^{\infty} 10^{-i} =$

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Proof: $\sum_{i=1}^{\infty} 10^{-i} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1,000} + \frac{1}{10,000} + \frac{1}{100,000} + ...$

$$= 0.1 + 0.01 + 0.001 + 0.0001 + 0.00001 + ...$$

Proof: $\sum_{i=1}^{\infty} 10^{-i} = \frac{1}{10} + \frac{1}{100} + \frac{1}{1,000} + \frac{1}{10,000} + \frac{1}{100,000} + ...$

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$= \frac{1}{9}$

$\therefore \sum_{i=1}^{\infty} 10^{-i} = \frac{1}{9}$

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$$\boxed{\sum_{i=1}^{\infty} \beta^{-i} = \frac{1}{\beta - 1} \text{ for any real number } \beta \text{ greater than } 1}$$

THANK YOU!