Undergraduate Mathemagic

A survey of surprising results and curiosities related to mathematics

Mohammad Behzad Kang

The University of Olivet
Math Seminar

November 15th, 2023

Outline

❖ Warm-Up

❖ Warm-Up

345

❖ Warm-Up

345 654

❖ Warm-Up

$$\begin{array}{r} 345 \\ \times \quad 21 \\ \hline \end{array}$$

$$\begin{array}{r} 654 \\ \times \quad 21 \\ \hline \end{array}$$

❖ Warm-Up

$$\begin{array}{r} 345 \\ \times \quad 21 \\ \hline \end{array} \qquad + \qquad \begin{array}{r} 654 \\ \times \quad 21 \\ \hline \end{array}$$

❶ + ❷

❖ Warm-Up

$$\begin{array}{r} 345 \\ \times \quad 21 \\ \hline \end{array} \qquad + \qquad \begin{array}{r} 654 \\ \times \quad 21 \\ \hline \end{array}$$

❶ + ❷

❶ + ❷

❖ Warm-Up

$$\begin{array}{r} 345 \\ \times \quad 21 \\ \hline \end{array} \qquad + \qquad \begin{array}{r} 654 \\ \times \quad 21 \\ \hline \end{array}$$

❶ + ❷

❶ + ❷ $= 21 \times 345 + 21 \times 654$

❖ Warm-Up

$$\begin{array}{r} 345 \\ \times \quad 21 \\ \hline \end{array}$$

❶

$+$

$$\begin{array}{r} 654 \\ \times \quad 21 \\ \hline \end{array}$$

❷

❶ $+$ ❷

$$= 21 \times 345 + 21 \times 654$$

$$= 21(345 + 654)$$

❖ Warm-Up

$$\begin{array}{r} 345 \\ \times \quad 21 \\ \hline \end{array}$$

❶

$+$

$$\begin{array}{r} 654 \\ \times \quad 21 \\ \hline \end{array}$$

❷

❶ $+$ ❷

$$= 21 \times 345 + 21 \times 654$$
$$= 21(345 + 654)$$
$$= 21(999)$$

❖ Warm-Up

$$\begin{array}{r} 345 \\ \times \quad 21 \\ \hline \end{array} \qquad + \qquad \begin{array}{r} 654 \\ \times \quad 21 \\ \hline \end{array}$$

❶ + ❷

$$\begin{aligned} & ❶ + ❷ \\ &= 21 \times 345 + 21 \times 654 \\ &= 21(345 + 654) \\ &= 21(999) \\ &= 21,000 - 21 \end{aligned}$$

❖ Warm-Up

$$\begin{array}{r} 345 \\ \times \quad 21 \\ \hline ❶ \end{array} \qquad + \qquad \begin{array}{r} 654 \\ \times \quad 21 \\ \hline ❷ \end{array}$$

❶ + ❷

$$\begin{aligned} &= 21 \times 345 + 21 \times 654 \\ &= 21(345 + 654) \\ &= 21(999) \\ &= 21,000 - 21 \\ &= 20,979 \end{aligned}$$

- Folding a sheet of paper

- Folding a sheet of paper
  - Thickness after n folds is $2^n \times 0.001$ cm.

- Folding a sheet of paper
  - Thickness after n folds is $2^n \times 0.001$ cm.
    - $n = 26$: Approximately 67,109 cm. thick

- Folding a sheet of paper
  - Thickness after n folds is $2^n \times 0.001$ cm.
    - $n = 26$: Approximately 67,109 cm. thick
    - $n = 46$: Approximately 7.04×10^{10} cm. thick

- Folding a sheet of paper
  - Thickness after n folds is $2^n \times 0.001$ cm.
    - $n = 26$: Approximately 67,109 cm. thick
    - $n = 46$: Approximately 7.04×10^{10} cm. thick
- Collatz conjecture

- How many yes/no questions are required to determine a mystery integer between 1 and 32?

- How many yes/no questions are required to determine a mystery integer between 1 and 32?

5

❖ How many yes/no questions are required to determine a mystery integer between 1 and 32?

5

☆ Binary search strategy (divide and conquer) is optimal here!

- How many yes/no questions are required to determine a mystery integer between 1 and 32?

5

- Binary search strategy (divide and conquer) is optimal here!

- Number of yes/no questions required to determine a mystery integer between 1 and n: $\lceil \log_2(n) \rceil$

Outline

- Birthday paradox:

- Birthday paradox: In a room of 23 people, there is over a 50% chance that at least two of them share the same birthday!

- Birthday paradox: In a room of 23 people, there is over a 50% chance that at least two of them share the same birthday!

- Monty Hall Problem

❖ Birthday paradox: In a room of 23 people, there is over a 50% chance that at least two of them share the same birthday!

❖ Monty Hall Problem – Suppose you're on a game show, and you're given the choice of three doors:

- Birthday paradox: In a room of 23 people, there is over a 50% chance that at least two of them share the same birthday!

- Monty Hall Problem – Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats.

- Birthday paradox: In a room of 23 people, there is over a 50% chance that at least two of them share the same birthday!

- Monty Hall Problem – Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats. You pick a door, say Door #1, and the host, who knows what's behind the doors, opens another door, say Door #3, to reveal a goat.

- Birthday paradox: In a room of 23 people, there is over a 50% chance that at least two of them share the same birthday!

- Monty Hall Problem – Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats. You pick a door, say Door #1, and the host, who knows what's behind the doors, opens another door, say Door #3, to reveal a goat. He then says to you, "Do you want to pick Door #2 instead?" Is it to your advantage to switch your choice of doors?

Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats. You pick a door, say Door #1, and the host, who knows what's behind the doors, opens another door, say Door #3, to reveal a goat. He then says to you, "Do you want to pick Door #2 instead?" Is it to your advantage to switch your choice of doors?

Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats. You pick a door, say Door #1, and the host, who knows what's behind the doors, opens another door, say Door #3, to reveal a goat. He then says to you, "Do you want to pick Door #2 instead?" Is it to your advantage to switch your choice of doors?

Behind Door 1	Behind Door 2	Behind Door 3	Result if you stay with Door #1	Result if you switch to Door #2
goat	car	goat	Win goat	Win car
car	goat	goat	Win car	Win goat
goat	goat	car	Win goat	Win car

Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats. You pick a door, say Door #1, and the host, who knows what's behind the doors, opens another door, say Door #3, to reveal a goat. He then says to you, "Do you want to pick Door #2 instead?" Is it to your advantage to switch your choice of doors?

Behind Door 1	Behind Door 2	Behind Door 3	Result if you stay with Door #1	Result if you switch to Door #2
goat	car	goat	Win goat	Win car
car	goat	goat	Win car	Win goat
goat	goat	car	Win goat	Win car

☆ In the three possible equally likely scenarios above, we win by switching doors in two out of the three!

Suppose you're on a game show, and you're given the choice of three doors: behind one door is a car; behind the others, goats. You pick a door, say Door #1, and the host, who knows what's behind the doors, opens another door, say Door #3, to reveal a goat. He then says to you, "Do you want to pick Door #2 instead?" Is it to your advantage to switch your choice of doors?

Behind Door 1	Behind Door 2	Behind Door 3	Result if you stay with Door #1	Result if you switch to Door #2
goat	car	goat	Win goat	Win car
car	goat	goat	Win car	Win goat
goat	goat	car	Win goat	Win car

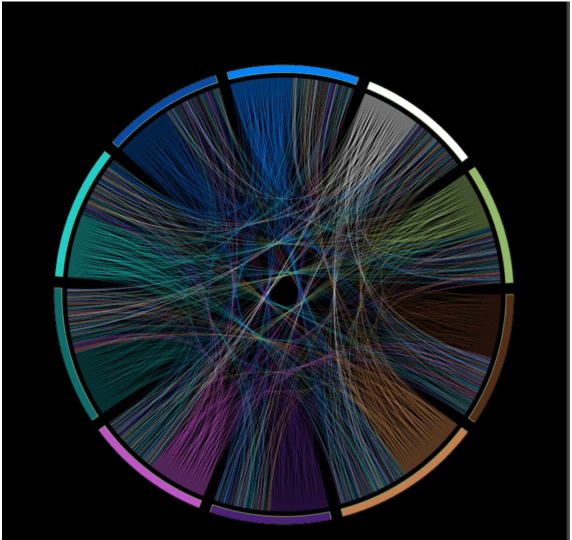
☆ In the three possible equally likely scenarios above, we win by switching doors in two out of the three!

Yes. Our chance of winning by switching doors is $\frac{2}{3}$!

Outline

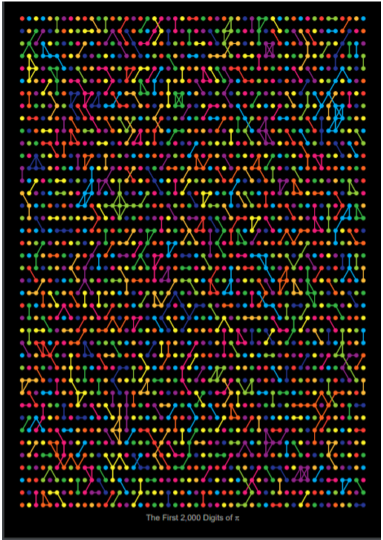
Visualizations of Pi

Visualizations of Pi

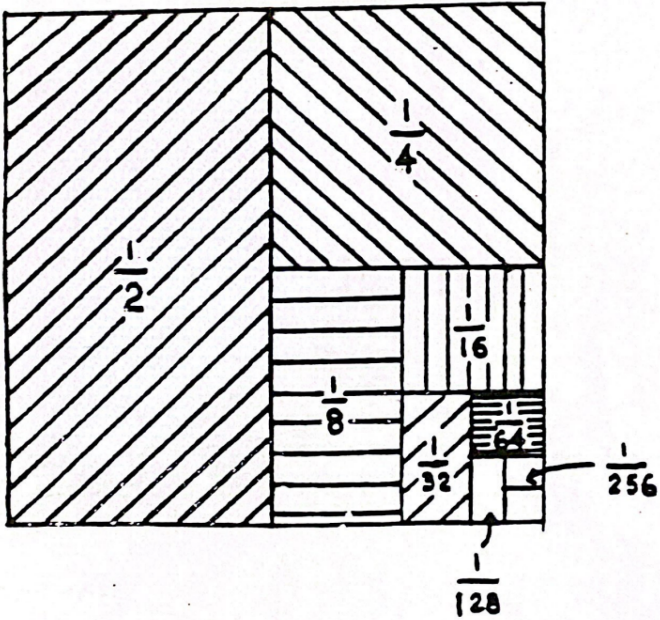


Visualizations of Pi

Visualizations of Pi



Outline


1/2
1/4
1/8
1/16
1/32
1/64
1/128
1/256

0

$$
\begin{aligned}
&0 \\
&= 0 + 0 + 0 + \cdots
\end{aligned}
$$

$$
\begin{aligned}
&0 \\
&= 0 + 0 + 0 + \cdots \\
&= 1 - 1 + 1 - 1 + 1 - 1 + \cdots
\end{aligned}
$$

$$
\begin{aligned}
&0\\
&= 0 + 0 + 0 + \cdots\\
&= 1 \text{ - } 1 + 1 \text{ - } 1 + 1 \text{ - } 1 + \cdots\\
&= 1 + \text{-}1 + 1 + \text{-}1 + 1 + \text{-}1 + \cdots
\end{aligned}
$$

$$
\begin{aligned}
& 0 \\
&= 0 + 0 + 0 + \cdots \\
&= 1 \text{ - } 1 + 1 \text{ - } 1 + 1 \text{ - } 1 + \cdots \\
&= 1 + \text{-}1 + 1 + \text{-}1 + 1 + \text{-}1 + \cdots \\
&= 1 + 0 + 0 + 0 + \cdots
\end{aligned}
$$

$$
\begin{aligned}
&0 \\
&= 0 + 0 + 0 + \cdots \\
&= 1 \text{ - } 1 + 1 \text{ - } 1 + 1 \text{ - } 1 + \cdots \\
&= 1 + \text{-}1 + 1 + \text{-}1 + 1 + \text{-}1 + \cdots \\
&= 1 + 0 + 0 + 0 + \cdots \\
&= 1
\end{aligned}
$$

$$
\begin{aligned}
&0\\
&= 0 + 0 + 0 + \cdots\\
&= 1 \text{ - } 1 + 1 \text{ - } 1 + 1 \text{ - } 1 + \cdots\\
&= 1 + \text{-}1 + 1 + \text{-}1 + 1 + \text{-}1 + \cdots\\
&= 1 + 0 + 0 + 0 + \cdots\\
&= 1
\end{aligned}
$$

$$0 = 1 \text{ ??}$$

Other Interesting Results

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots$

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \quad = \frac{\pi^2}{6}$

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \quad = \frac{\pi^2}{6}$
- Choose a random number in $[0, 1]$ and record its value.

Other Interesting Results

- $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\cdots \quad = \frac{\pi^2}{6}$
- Choose a random number in $[0,1]$ and record its value. Do this again and add the second number to the first number.

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \quad = \frac{\pi^2}{6}$
- Choose a random number in $[0, 1]$ and record its value. Do this again and add the second number to the first number. Keep doing this until the sum of all of your numbers exceeds 1.

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \quad = \frac{\pi^2}{6}$
- Choose a random number in $[0, 1]$ and record its value. Do this again and add the second number to the first number. Keep doing this until the sum of all of your numbers exceeds 1. The expected value of the amount of random numbers needed to accomplish this is precisely...

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \quad = \frac{\pi^2}{6}$
- Choose a random number in $[0, 1]$ and record its value. Do this again and add the second number to the first number. Keep doing this until the sum of all of your numbers exceeds 1. The expected value of the amount of random numbers needed to accomplish this is precisely... e!

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \quad = \frac{\pi^2}{6}$
- Choose a random number in $[0, 1]$ and record its value. Do this again and add the second number to the first number. Keep doing this until the sum of all of your numbers exceeds 1. The expected value of the amount of random numbers needed to accomplish this is precisely... *e*!
- Hairy ball theorem:

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \quad = \frac{\pi^2}{6}$
- Choose a random number in $[0, 1]$ and record its value. Do this again and add the second number to the first number. Keep doing this until the sum of all of your numbers exceeds 1. The expected value of the amount of random numbers needed to accomplish this is precisely... e!
- Hairy ball theorem: You can't comb the hairs flat on a coconut without creating a cowlick!

Other Interesting Results

- $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \cdots \quad = \frac{\pi^2}{6}$
- Choose a random number in $[0, 1]$ and record its value. Do this again and add the second number to the first number. Keep doing this until the sum of all of your numbers exceeds 1. The expected value of the amount of random numbers needed to accomplish this is precisely... e!
- Hairy ball theorem: You can't comb the hairs flat on a coconut without creating a cowlick!
- Solution to Seven Bridges of Königsberg Problem

Thank you for listening!